

Bagatelle as the inspiration for Galton's Quincunx

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This paper considers the question of how Francis Galton came to devise the quincunx, a pinboard that simulates the effect of a large number of Bernoulli trials to yield an empirical normal curve. It suggests that the likely inspiration for the design was some variant of bagatelle. The argument forms a small part of an Open University doctoral thesis (May 2005), which revealed the extent of the support Galton received from George Howard Darwin as he developed the concept of statistical regression.

The Friday evening Royal Institution Discourses, established by Humphry Davy in 1826, were delivered by scientists renowned for their presentational skills to formally dressed audiences, who expected the lecture to be enhanced with experiments. At such an event on 27 February 1874, Francis Galton (1822–1911) revealed a radical approach to statistical data in which ranks replaced direct measurement. The method would circumvent the problem of taking certain physical measurements when the subject was uncomfortable or openly hostile (as was his experience with the tribes people of southwest Africa in the early 1850s) and would provide surrogate data for psychometric studies in which there was often no scale for reference, making absolute measurement impossible.

However, before he could unveil the new methods on that night, Galton had to find a way to bridge the huge gulf in understanding between himself and his audience, some of whom had scant knowledge of what we now refer to as the normal distribution and which he called the 'law of frequency of error' or the 'law of deviation'. Without such understanding the method of ranks would remain a mystery to all but a few in attendance. Galton's solution was to focus on the communication of his ideas, and very much in the spirit of the Royal Institution, he demonstrated a vital result using a simulation device in the form of a pinboard. In 1873, he had commissioned Tisley and Spiller, makers of scientific instruments, to construct a device for demonstrating the connection between Bernoulli trials, the binomial law, and the law of frequency (see Figure 1).¹ He thought of it as a machine for illustrating the 'Cause of the Curve of Frequency', one that 'mimics in a very pretty way the conditions on which Deviation depends' (Galton 1889, 63). It was not alluded to specifically in the report of the talk but Galton later referred to 'an apparatus that I employed three years ago in this very theatre, to illustrate . . . points connected with the law of deviation' (Galton 1877).

The apparatus consisted of a pinboard in a glass case and was dubbed a 'quincunx' by Galton because it reminded him of the design on the Roman coin of

¹ According to Julian Holland of the Macleay Museum at the University of Sydney, Tisley and his associate, George Spiller, were scientific instrument makers who, at their premises in Brompton Road, 'sought to capitalize on the expansion of science education in England after 1871' (Holland 2000).

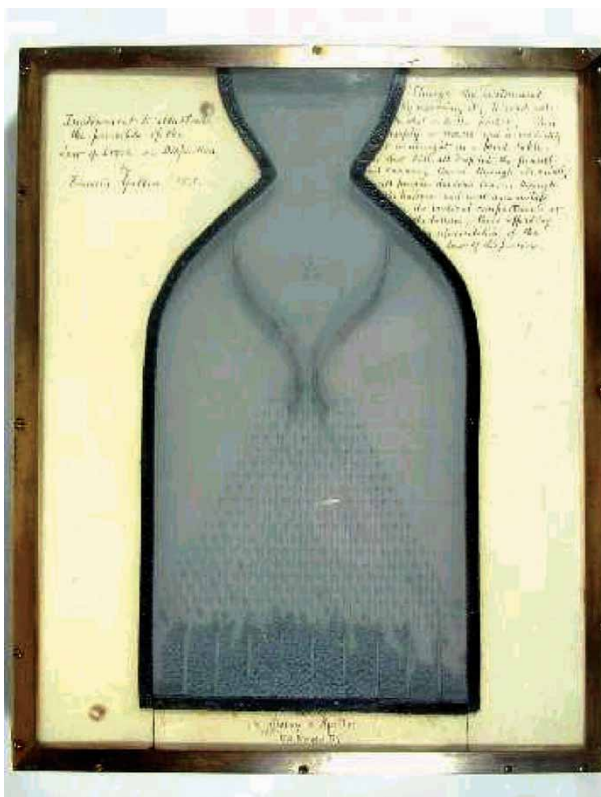


Figure 1. The original quincunx, made by Tisley & Spiller in 1873

that denomination—four dots arranged in a square, with a fifth at the centre.² Lead shot was poured in through the top and as each particle descended it was deflected randomly to the left or right as it met a pin. Every deflection was independent of every other and represented both a Bernoulli trial and, more meaningfully, one amongst the multiplicity of barely significant causes that combine to produce human variability. The instructions for the use of the quincunx were written by Galton above and to the right of the device:

Charge the instrument by reversing it, to send all the shot into the pocket. Then sharply re-reverse and immediately set it upright on a level table. The shot will all drop into the funnel, and running thence through its mouth, will pursue devious courses through the harrow and will accumulate in the vertical compartments at the bottom, there affording a representation of the law of dispersion.

In Galton's first published description of the quincunx in action, in *Natural Inheritance*, he wrote:

The cascade issuing from the funnel broadens as it descends, and, at length, every shot finds itself caught in a compartment immediately after freeing itself from

²The word derives from the Latin *quinque*, five and *uncia*, a twelfth. The quincunx was a symbol used by the Romans for five-twelfths of an *as*, a copper coin. The word had been used in England since the seventeenth century to describe the arrangement of trees in an orchard.

the last row of pins. The outline of the columns of shot that accumulate in the successive compartments approximate to the Curve of Frequency ... and is closely of the same shape however often the experiment is repeated. The outline of the columns would become nearly identical with the Normal Curve of Frequency, if the rows of pins were much more numerous, the shot smaller, and the compartments narrower; also if a larger quantity of shot was used.³ (Galton 1889, 64)

As Stephen Stigler commented in his incomparable *History of statistics*, ‘as a device for illustrating lectures [the quincunx] must have been marvelously effective’, and it gave the audience a better feel for the increasing consonance between the binomial law and the law of frequency of error as the lead shot assembled in the receiving slots (Stigler 1986, 277).⁴

But Galton gave no clue as to the inspiration for the pinboard demonstration and neither is there anything in the historical literature. There may be a clue, however, in the only drawing of the simple quincunx that Galton published (Figure 2). It is to be found in his *Natural inheritance* in conjunction with the description of the quincunx quoted above. Note that the arrangement of pins lies within an overall rectangular design, which would allow for some shot to reach the extreme right-hand or left-hand margins and be cushioned back by the frame. This would compromise the outcome of the experiment if actually used. This is not a feature of the quincunx demonstrated by Galton.

The diagram is reminiscent of the sloping boards used for at least two variants of the game of bagatelle. Bagatelle originated in France around the reign of Louis XIV (1636–1715).⁵ The game of ‘Nine Holes’, in which small balls were rolled into holes labelled 1 to 9, became popular in England at much the same time. As Robbie Bell (1876, 16) observed, ‘a hundred and fifty years later this form of Nine Holes had developed into bagatelle’, that is, into the popular nineteenth-century table game of bagatelle. According to the *Oxford English dictionary*, the first reference to the game of bagatelle in English is to be found in the *Post Office directory*, published in 1819. Here, John Thurston of Catherine Street, just off the Strand, is listed as a manufacturer of bagatelle boards. In fact, Thurston, who was better remembered for his production of billiard tables, was already the leading maker of bagatelle boards, and production at his workshop increased until 1845, the year when playing bagatelle for money was restricted under a new Gaming Act.

By this time, the bagatelle table had become a feature of both the English inn and many private houses. In his first novel, *The Pickwick papers*, written as Victoria came

³In 1892, with the help of Ludwig Boltzmann, the topologist and group theorist, Walther Franz Anton Dyck (1856–1934) organized the first international exhibition of mathematics and physics instruments in Munich, actually postponed to the following year because of a cholera outbreak. The catalogue contains a description in German of the quincunx, submitted by Galton. See Galton. 1892/1994.

⁴A photograph of the original quincunx was given by Stigler (1986, 277), in which the empirical distribution is more distinctly ‘normal’ than the figure in this article suggests. Part of the problem in capturing the image well is the fact that there is a coating of lead dust on the inside of the glass.

⁵It took its name from what was originally a ‘trifling’ residence outside Paris, containing a *salon de jeu*, which in the eighteenth century attracted the gambling fraternity of the French aristocracy.

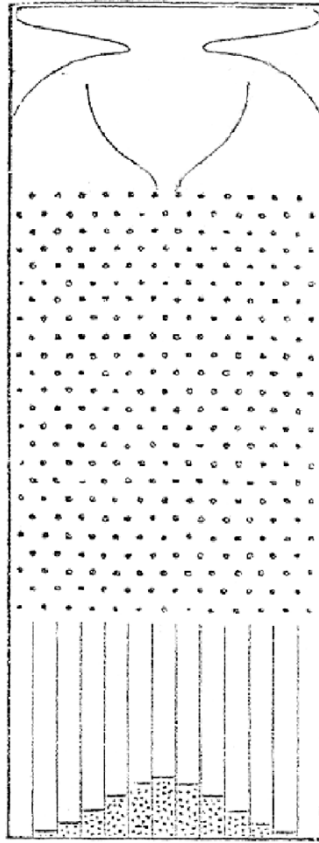


Figure 2. Galton's drawing of a quincunx, 1889

to the throne, Charles Dickens painted a vivid picture of members of the Pickwick Club relaxing at the bagatelle table in the Peacock Tavern:

Mr. Tupman and Mr. Snodgrass were in a great measure cast upon their own resources. Taking but little interest in public affairs, they beguiled their time chiefly with such amusements as the Peacock afforded, which were limited to a bagatelle-board in the first floor, and a sequestered skittle-ground in the back yard. In the science and nicety of both these recreations, which are far more abstruse than ordinary men suppose, they were gradually initiated by Mr. Weller, who possessed a perfect knowledge of such pastimes.⁶ (Dickens 1836–37)

Tupman and Snodgrass probably played the most common form of bagatelle, on a table of upwards of six feet, with its distinctive numbered holes or cups. But there were variants of the game dating back to the early years of the century. One form of bagatelle played in this early period featured a board with characteristic Regency *chinoiserie* and a definite quincuncial pin arrangement, and went by the 'evocative name' of 'The Rocks of Scilly' (Opie *et al.* 1989, 158–159). An example is depicted

⁶The book was published in monthly instalments from March 1836 until November 1837. The reference to bagatelle, in the introductory paragraph of chapter 14, is the earliest in English literature.



Figure 3. Rocks of Scilly board and baton, c. 1810

in Figure 3. The game is described in John Strutt's popular book, *The sports and pastimes of the people of England*:

This diversion requires a table oblong in its form, and curved at the top, which is more elevated than the bottom. There is a hollow trunk affixed to one side, which runs nearly the whole length of the table, and is open at both ends. The balls are put in singly at the bottom, and driven through it by the means of a round baton of wood. When a ball quits the trunk it is impelled by its own gravity towards

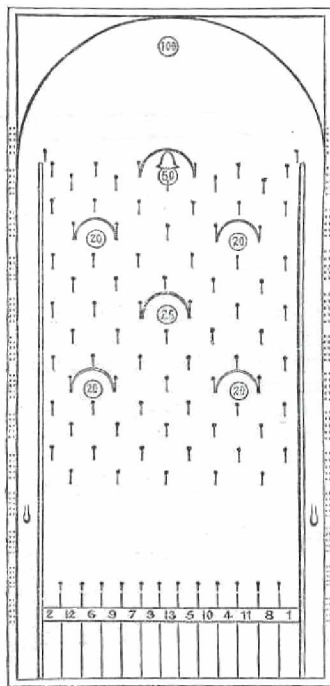


Figure 4. Cockamaroo, as drawn in 1850

the lower part of the table, where there are arches ... but it is frequently interrupted in its descent by wires inserted at different distances upon the table, which alter its direction, and often throw it entirely out of the proper track.⁷ (Strutt 1801/1833, 301)

There was a second quincuncial version, called ‘Cockamaroo’ or ‘Russian Bagatelle’. A drawing and description of the Cockamaroo table was given in Henry Bohn’s *Hand book of games* of 1850, and is shown in Figure 4. It was played on a table of just four feet six inches in length. Shots were taken alternately from the left and right channels by the two players, the ivory balls rolling down the inclined table and deflecting off the 1½ inch brass pins until they landed either in one of the seven numbered cups or in the numbered receptacles at the bottom. In 1850, the best tables were made by Thurston and available at a price of fifteen guineas (Bohn 1850, 613–614).

The Rocks of Scilly and Cockamaroo are very similar games and it seems likely that the latter developed from the former. The Cockamaroo table featured an arch over six of the seven cups and a bell, which if rung doubled the score on that shot. Another difference lay in the number of pins. The earlier version had 12 rows of four or five pins; the later had 14 rows of five or six pins. The undated pin bagatelle table shown in Figure 5 appears to have 13 rows of seven or eight pins.

⁷I am advised by David Singmaster that the book went through numerous editions and reprintings, the last as recent as 1969. The description is from the 1833 edition. Note that since the ‘baton’ would slide up the ‘trunk’ taking the ball with it, the skill lay in pushing just hard enough to persuade the ball to turn under gravity’s influence at the optimum point on the rim.



Figure 5. A nineteenth-century pin bagatelle board (exact date unknown)

From top to bottom, the scoring cups in the surface of the Rocks of Scilly board are marked 10, 30, 20 twice, 50, and 20 twice again. The corresponding cups in the Cockamaroo board are identically positioned but are marked 100, 50, 20 twice, 25 and 20 twice again. The unnamed pin bagatelle board has four holes and two further holes with two sliding plates but no numbered receiving slots at the bottom.

As well as the quincunxial pin arrangements, the ordering of the numbered receptacles is interesting and could be used to support the notion of a link between variants of bagatelle and the design of the quincunx, or the reverse. The sequences from left to right on the two boards are:

Rocks of Scilly	7	3	5	2	8	4	1	6					
Cockamaroo	2	12	6	9	7	3	13	5	10	4	11	8	1

There appears to be a randomness about each of them, possibly to reduce the effect of the tendency towards mediocrity. However, the positioning (on the Cockamaroo board) of the lowest scoring receptacles at the periphery and the highest scoring in the middle provides evidence to the contrary. Overall, the sequences are reminiscent of the numbering on a darts board, where a tilt towards randomness is countered by considerations of symmetry and the strategy of positioning low-scoring cells adjacent to high-scoring cells in order to reduce the element of chance.⁸

Billiards, bagatelle's more popular cousin, was certainly played by Galton, by his cousin Charles Darwin and by Charles' son, George Darwin, who at the time of the Royal Institution talk was very much under Galton's influence. Galton probably

⁸The numbering on a darts board postdates the sequences on these pin bagatelle boards. It was devised in 1896 by Brian Gamlin, a Bury carpenter.

played the game at the Athenaeum from time to time. Towards the end of his life he recalled that it was Herbert Spencer's

custom for many afternoons to spend an hour or two of rest in the old smoking room of the Athenaeum Club, strolling into an adjoining compartment for a game of billiards when the table was free. Day after day on those afternoons I enjoyed brief talks with him... (Galton 1907, 165)⁹

But there was not too much skill to Galton's game, as he commented in a discussion with Max Müller, published in *Nature* in 1887:

It is well known at billiards that some persons play much more 'with their heads' than others. I am but an indifferent player; still, when I do play, I think out the best stroke as well as I can, but not in words. I hold the cue with nascent and anticipatory gesture, and follow the probable course of the ball from cushion to cushion with my eye before I make the stroke... (Galton 1887)

Charles Darwin bought a billiard table from Hopkins and Stephens of Covent Garden in 1859.¹⁰ George took to the game with youthful enthusiasm and demonstrated a certain flair, his father reporting within weeks that 'you never saw anything like Georgy & the Billiards; I think on Saturday he played for 10 hours'; also that 'George has a nice notion of playing'.¹¹

Such was the popularity of bagatelle in Galton's day, it is highly likely that he would have been familiar with it as well. Whether he was aware of the variant forms of the game that featured quincuncial arrangements of sturdy brass pins and which evolved in the twentieth century into pin-ball, is not known; neither do we know if Galton possessed a bagatelle table or board of any kind. Nevertheless, there is sufficient *prima facie* evidence to suggest that the primary influence on the design of the quincunx was some variant form of bagatelle, probably The Rocks of Scilly or Cockamaroo.¹²

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⁹This was in the Herbert Spencer Lecture, delivered at Oxford University on 5 June 1907.

¹⁰Charles Darwin, letter to George Darwin, 24 [February 1859], Cambridge University Library, DAR 210.6:37. The table which cost a little under £54, was installed at Down on the day the letter was written. Hopkins and Stephens were nephews of John Thurston.

¹¹Charles Darwin, letter to William Erasmus Darwin, 14 [March 1859], Cambridge University Library, DAR 210.6:40.

¹²I am indebted to David Singmaster for finding references to The Rocks of Scilly and Cockamaroo.

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